Algebra 2
Conic Sections
Hyperbolas

Determine the equation of each hyperbola using the description given.

1. What is the equation of the hyperbola with vertices (0, 5) and (0, -5) and co-vertices at (9, 0) and (-9, 0)?

The center of the hyperbola is found by finding the midpoint of the vertices, which is (0, 0).
Therefore, h = 0 and k = 0.
This is a vertical hyperbola because the vertices are vertical. Therefore, we can find the value of b by setting (0, 0 + a) = (0, 5). This results in a value of a = 5.
Similarly, we can find the value of a by setting (0 + b, 0) = (9, 0). This results in a value of b = 9.
The equation for this hyperbola is $\frac{y^2}{25}-\frac{x^2}{81}=1$.

2. What is the equation of the hyperbola with vertices (7, 0) and (-7, 0) and co-vertices at (0, 4) and (0, -4)?

The center of the hyperbola is found by finding the midpoint of the vertices, which is (0, 0).
Therefore, h = 0 and k = 0.
This is a horizontal hyperbola because the vertices are horizontal. Therefore, we can find the value of a by setting (0 + a, 0) = (7, 0). This results in a value of a = 7.
Similarly, we can find the value of b by setting (0, 0 + b) = (0, 4). This results in a value of b = 4.
The equation for this hyperbola is $\frac{x^2}{49}-\frac{y^2}{16}=1$.

3. What is the equation of the hyperbola with foci (6, 0) and (-6, 0) and co-vertices at (0, 3) and (0, -3)?

The center of the hyperbola is found by finding the midpoint of the co-vertices, which is (0, 0).
Therefore, h = 0 and k = 0.
This is a horizontal hyperbola because the vertices are horizontal. Therefore, we can find the value of c by setting (0 + c, 0) = (6, 0). This results in a value of c = 6.
Similarly, we can find the value of b by setting (0, 0 + b) = (0, 3). This results in a value of b = 3.
To find the value of a, the relationship is:
$a^2 + b^2 = c^2$
$a^2 + 3^2 = 6^2$
$a^2 + 9 = 36$
$a^2 = 27$
$a = \sqrt{27}$
The equation for this hyperbola is $\frac{x^2}{27}-\frac{y^2}{9}=1$.

4. What is the equation of the hyperbola with foci (0, 8) and (0, -8) and co-vertices at (4, 0) and (-4, 0)?

The center of the hyperbola is found by finding the midpoint of the co-vertices, which is (0, 0). Therefore, h = 0 and k = 0.
This is a vertical hyperbola because the vertices are vertical. Therefore, we can find the value of c by setting (0, 0 + c) = (0, 8). This results in a value of c = 8.
Similarly, we can find the value of a by setting (0 + b, 0) = (4, 0). This results in a value of b = 4.
To find the value of a, the relationship is:
$a^2 + b^2 = c^2$

$a^2 + 4^2 = 8^2$
$a^2 + 16 = 64$
$a^2 = 48$
$a = \sqrt{48}$

The equation for this hyperbola is $\frac{y^2}{48} - \frac{x^2}{16} = 1$.

5. What is the equation of the hyperbola with foci (0, 5) and (0, -5) and vertices at (0, 3) and (0, -3)?

The center of the hyperbola is found by finding the midpoint of the vertices, which is (0, 0).
Therefore, h = 0 and k = 0.
This is a vertical hyperbola because the vertices are vertical. Therefore, we can find the value of c by setting (0, 0 + c) = (0, 5). This results in a value of c = 5.
Similarly, we can find the value of a by setting (0, 0 + a) = (0, 3). This results in a value of a = 3.
To find the value of b, the relationship is:
$a^2 + b^2 = c^2$
$3^2 + b^2 = 5^2$
$9 + b^2 = 25$
$b^2 = 16$
$b = 4$

The equation for this hyperbola is $\frac{y^2}{9} - \frac{x^2}{16} = 1$.

6. What is the equation of the hyperbola with foci (8, 0) and (-8, 0) and vertices at (5, 0) and (-5, 0)?

The center of the hyperbola is found by finding the midpoint of the vertices, which is (0, 0).
Therefore, h = 0 and k = 0.
This is a horizontal hyperbola because the vertices are horizontal. Therefore, we can find the value of c by setting (0 + c, 0) = (8, 0). This results in a value of c = 8.
To find the value of b, the relationship is:
$a^2 + b^2 = c^2$
$5^2 + b^2 = 8^2$
$25 + b^2 = 64$
$b^2 = 39$
$b = \sqrt{39}$

The equation for this hyperbola is $\frac{x^2}{25} - \frac{y^2}{39} = 1$.

7. What is the equation of the hyperbola with foci at (2, 4) and (2, -8) and vertices at (2, 2) and (2, -6)?

The center of the hyperbola is found by finding the midpoint of the vertices, which is (2, -2).
Therefore, h = 2 and k = -2.
This is a vertical hyperbola because the vertices are vertical. Therefore, we can find the value of c by setting (2, -2 + c) = (2, 4). This results in a value of c = 6.
Similarly, we can find the value of a by setting (0, -2 + a) = (2, 2). This results in a value of a = 4.
To find the value of b, the relationship is:
$a^2 + b^2 = c^2$
$4^2 + b^2 = 6^2$
$16 + b^2 = 36$
$b^2 = 20$
$b = \sqrt{20}$

The equation for this hyperbola is $\frac{(y+2)^2}{16}-\frac{(x-2)^2}{20}=1$.

8. What is the equation of the hyperbola with foci at (0, 6) and (6, 6) and vertices at (1, 6) and (5, 6)?

The center of the hyperbola is found by finding the midpoint of the vertices, which is (3, 6).
Therefore, h = 3 and k = 6.
This is a horizontal hyperbola because the vertices are horizontal. Therefore, we can find the value of c by setting (3 + c, 6) = (6, 6). This results in a value of c = 3.
Similarly, we can find the value of a by setting (3 + a, 6) = (5, 6). This results in a value of a = 2.
To find the value of b, the relationship is:
$a^2 + b^2 = c^2$
$2^2 + b^2 = 3^2$
$4 + b^2 = 9$
$b^2 = 5$
$b = \sqrt{5}$

The equation for this hyperbola is $\frac{(x-3)^2}{4}-\frac{(y-6)^2}{5}=1$.

9. What is the equation of the hyperbola with foci (-2, 5) and (6, 5) and co-vertices at (2, 8) and (2, 2)?

The center of the hyperbola is found by finding the midpoint of the vertices, which is (2, 5).
Therefore, h = 2 and k = 5.
This is a horizontal hyperbola because the vertices are horizontal. Therefore, we can find the value of c by setting (2 + c, 5) = (6, 5). This results in a value of c = 4.
Similarly, we can find the value of b by setting (2, 5 + b) = (2, 8). This results in a value of b = 3.
To find the value of a, the relationship is:
$a^2 + b^2 = c^2$
$a^2 + 3^2 = 4^2$
$a^2 + 9 = 16$
$a^2 = 7$
$a = \sqrt{7}$

The equation for this hyperbola is $\frac{(x-2)^2}{7}-\frac{(y-5)^2}{9}=1$.

10. What is the equation of the hyperbola with foci (4, -2) and (4, -8) and co-vertices at (4 + $\sqrt{5}$, -5) and (4 – $\sqrt{5}$, -5)?

The center of the hyperbola is found by finding the midpoint of the vertices, which is (4, -5).
Therefore, h = 4 and k = -5.
This is a vertical hyperbola because the vertices are vertical. Therefore, we can find the value of c by setting (4, -5 + c) = (4, -2). This results in a value of c = 3.
Similarly, we can find the value of b by setting (4 + b, -5) = (4 + $\sqrt{5}$, -5). This results in a value of b = $\sqrt{5}$.
To find the value of a, the relationship is:
$a^2 + b^2 = c^2$
$a^2 + (\sqrt{5})^2 = 3^2$
$a^2 + 5 = 9$
$a^2 = 4$
a = 2

The equation for this hyperbola is $\frac{(y+5)^2}{4}-\frac{(x-4)^2}{5}=1$.