

Course: Algebra 2
Unit: Conic Sections
Section: Hyperbolas

Example: Graphing a Hyperbola Centered at the Origin

Problem:

Graph the hyperbola y^2 divided by 16 minus x^2 divided by 36 equals 1.
Include the vertices, the co-vertices, the asymptotes, and the foci.

Solution:

Since the y term is first, this equation represents a vertical transverse axis. The vertices are at the points $(0, a)$ and $(0, \text{negative } a)$.

Notice that the equation tells us that a^2 equals 16. This means that a equals 4. The vertices, then, are at the points $(0, 4)$ and $(0, \text{negative } 4)$. Plot these points.

The co-vertices are at the points $(b, 0)$ and $(\text{negative } b, 0)$.

Notice that the equation tells us that b^2 equals 36. This means that b equals 6. The co-vertices, then, are at the points $(6, 0)$ and $(\text{negative } 6, 0)$. Plot these points.

Now we will use these four points to outline a rectangle.

Using this rectangle as a guide, draw lines through both diagonals.

It is easy to see once you have graphed the asymptotes, that the slope of the positive sloped asymptote is positive $\frac{4}{6}$ and the slope of the negative sloped asymptote is negative $\frac{4}{6}$. These values correspond to the a and b values. The equations of these lines are y equals plus or minus two-thirds.

The last piece of information we need is the foci. Recall that $a^2 + b^2 = c^2$. Using the a and b values, we can write the equation $4^2 + 6^2 = c^2$. Solving this step by step, the result is $c = 2\sqrt{13}$. The foci are at the points $(0, 2\sqrt{13})$ and $(0, \text{negative } 2\sqrt{13})$.

Plot these points.

The very last step is to graph the actual hyperbola. Using the vertices as starting points and the asymptotes as guides to how wide the hyperbola should be, graph both parts.

Notice that the hyperbola opens up and down, with the foci in the middle of each side. Notice also, that each half uses the asymptotes as guides to how wide they are.