

Course: Algebra 2
Unit: Conic Sections
Section: Hyperbolas

Example: Graphing a Hyperbola Centered at (h, k)

Problem:

Graph the hyperbola $x - 2$ quantity squared divided by 25 minus $y + 1$ quantity squared divided by 16 equals 1.

Include the center, the vertices, the co-vertices, and the foci.

Solution:

The center of the hyperbola will be at the point h, k . Using the general equation for a parabola, we can see that the center is at the point 2, negative 1. Plot this point.

Since the x term is first, this equation represents a horizontal transverse axis. The vertices are at the points $h - a, k$ and $h + a, k$.

Notice that the equation tells us that a squared equals 25. This means that a equals 5. The vertices are at the points negative 3, negative 1 and 7, negative 1. Plot these points.

The co-vertices are at the points $h, k - b$ and $h, k + b$.

Notice that the equation tells us that b squared equals 16. This means that b equals 4. The co-vertices are at the points 2, negative 5 and 2, 3. Plot these points.

Now we will use these four points to outline a rectangle.

Using this rectangle as a guide, draw lines through both diagonals.

It is easy to see once you have graphed the asymptotes, that the slope of the positive sloped asymptote is positive 4 over 5 and the slope of the negative sloped asymptote is negative 4 over 5. These values correspond to the a and b values. It would be fairly easy to use our Algebra skills to find the equations of these lines, but at this point, it is not necessary. Simply having the asymptotes graphed is enough.

The last piece of information we need is the foci. Recall that a squared plus b squared equals c squared. Using the a and b values, we can write the equation 5 squared plus 4 squared equals c squared. Solving this step by step, the result is c equals square root of 41. The foci are at the points 2 minus square root of 41, negative 1 and 2 plus square root of 41, negative 1.

Plot these points.

The very last step is to graph the actual hyperbola. Using the vertices as starting points and the asymptotes as guides to how wide the hyperbola should be, graph both parts.

Script

Notice that the hyperbola opens right and left, with the foci in the middle of each side. Notice also, that each half uses the asymptotes as guides to how wide they are.