

**Course: Algebra 2**  
**Unit: Conic Sections**  
**Section: Ellipses**

**Example: Graphing an Ellipse with a Center (h, k)**

**Problem:**

Graph the ellipse  $x$  minus 3 quantity squared divided by 9 plus  $y$  plus 1 quantity squared divided by 25 equals 1.

Once the graph is made, find the foci, the major axis and the minor axis.

**Solution:**

The first step to graphing an ellipse is to graph its center. Recall from the general formula that the center is at the point  $h, k$ . In this case, the center is at the point three, negative 1. Plot that point.

Next, we need to graph the vertices and the co-vertices. Recall from the general formula for an ellipse, that if the number under the  $y$  is greater than the number under the  $x$ , it is a vertical ellipse, the major axis is parallel to the  $y$ -axis. This means that the vertices will be directly above and below the center.

To find how far the vertices are away from the center, we need to know the value of  $a$ .  $A$  squared equals 25,  $a$  equals 5. The vertices have the same  $x$ -value as the center, but different  $y$ -values. The points are  $h, k$  plus or minus  $a$ . For this ellipse, this is the points 3, negative 1 plus or minus 5. This gives us the points 3, 4 and 3 negative 6.

Graph these two vertices on the coordinate axis.

To find how far the co-vertices are away from the center, we need to know the value of  $b$ .  $B$  squared equals 9,  $b$  equals 3. The vertices have the same  $y$ -value as the center, but different  $x$ -values. The points are  $h$  plus or minus  $b, k$ . For this ellipse, this is the points 3 plus or minus 3, negative 1. This gives us the points 6, negative 1 and 0, negative 1.

Graph these two co-vertices on the coordinate axis.

Now, we have enough points to graph an accurate ellipse.

The problem also asks us to find the foci. In order to do that, we first need to know the value of  $c$ .  $C$  squared equals  $a$  squared minus  $b$  squared.  $C$  squared equals 5 squared minus 3 squared,  $c$  squared equals 25 minus 9, which is 16. This means  $c$  equals 4.

The foci are in the same direction as the vertices. To find them, we use the point  $h, k$  plus or minus  $c$ . In this case, those points are 3, negative 1 plus or minus 4, which gives us the points 3, 3 and 3, negative 5

These points can be graphed.

The major axis is the length across the ellipse in the longer direction. In this case, that is in the vertical direction.  $2a$  equals 2 times 5 equals 10. You can see from the graph that the longest distance across the graph is 10 units.

The minor axis is the length across the ellipse in the shorter direction. In this case, that is in the horizontal direction.  $2b$  equals 2 times 3 equals 6. You can see from the graph that the shorter distance across the graph is 6 units.