

Algebra 2
Unit: Geometry
Section: Geometry of Circles

Example: Tangents and Secants of Circles

Problem:

Shown below is circle C, with tangent lines AB and DB. Given the following information, set up two equations and solve for x and y.

AB equals five x minus two y.

DB equals forty.

The measure of arc AED equals four x plus six y.

The measure of arc AD equals two x plus two y.

The measure of angle B equals eighty degrees.

Solution:

Let's start with the information about the tangent lines. One property of circles is that two tangent lines to a circle, drawn from the same point outside the circle, have equal measure.

This means that the length of AB equals the length of DB.

Based on that and the information given, we can write the equation five x minus two y equals forty.

Now, let's look at the arcs. Another property of circles is that the angle formed outside the circle will be half the difference of the two intercepted arcs.

The measure of angle B equals half the difference of the measures of arc AED and arc AD.

Here we must be very careful to enter the information given correctly. Eighty equals one-half the quantity four x plus six y minus the quantity two x plus two y.

This simplifies to eighty equals one-half the quantity two x plus four y, which simplifies further to eighty equals x plus two y.

Now we have two equations with two variables. We can solve these using either substitution or elimination.

Five x minus two y equals forty.

X plus two y equals eighty.

These equations can be added together and the y terms will eliminate.

This leaves us with six x equals one hundred twenty.

X equals twenty.

We can substitute the value we got for x into one of the equations. For simplicity, let's use the equation $x + 2y = 80$. $20 + 2y = 80$. $2y = 60$. $y = 30$.

It would be easy to substitute these values for x and y to see if they make arcs and tangent lines that meet the properties of circles we have learned.