Algebra 2
Unit: Systems of Equations and Inequalities
Section: Systems of Equations with Three Variables

Example: Solving Systems of Equations with Three Variables Using Matrices

Problem

Solve this system of equations. x plus y plus z equals two. Two x equals y plus four. x plus three z equals ten.

Solution

Step 1. Make sure that all equations are in the correct form. This means that all variables must be on the left side and all constants are on the right side.

The second equation, two x equals y plus 4 should be changed to two x minus y equals 4.

Step 2. Set up the matrix equation. When you do this, be sure to insert zeros for the variables that are not in each equation and be sure to put the coefficients in the correct order.

Notice that the first number in each row, one, two, one corresponds to the coefficient of the x term in each equation. Notice that the second number in each row, one, negative one, zero corresponds to the coefficient of the y term in each equation. There is a zero in the last row because the third equation does not contain a y term. Notice that the third number in each row, one, zero, three corresponds to the coefficient of the z term in each equation. There is a zero in the second row because the second equation does not contain a z term. The three by one matrix on the right side contains the constant values in each equation, two, four, ten.

Step 3. Find the inverse matrix of A. You do not need to be able to do this algebraically.

Using any one of the online tools given to you or your own graphing calculator, you get the inverse matrix of A. The first row is three-eighths, three-eighths, negative one-eighth; the second row is three-fourths, negative one-fourth, negative one-fourth; and the third row is negative one-eighth, negative one-eighth, three-eighths.

Step 4. Multiply the inverse of A times B.

The result when you multiply the inverse of A times B is the three by one matrix with terms: one, negative two, three.

The answer then, is the ordered triple x equals 1, y equals negative 2 and z equals 3. It is always a good idea to check your answer by substituting these values into the original equations.