

**Tutorial: Angles of Lines that Intersect Outside a Circle****Slide 1:**

In this tutorial we are going to discuss how to solve problems involving lines that intersect outside a circle.

**Slide 2:**

Tangents and secants.

A tangent is a line in the same plane as a circle that intersects the circle in exactly one point, called the point of tangency.

A secant is a line that intersects a circle at exactly two points. The following are possible scenarios of tangents and secants intersecting outside a circle.

Scenario 1. Two secants.

Measure of angle E is equal to one half the quantity measure of arc AC minus measure of arc BD.

Scenario 2. Secant Tangent.

Measure of angle B is equal to one half the quantity measure of arc AD minus measure of arc AC.

Scenario 3. Two tangents.

Measure of angle B is equal to one half the quantity measure of arc AEC minus measure of arc AC.

**Slide 3:**

Example.

The measure of arc BD is twenty six degrees.

The measure of arc AC is eighty eight degrees.

Find the measure of angle E.

This is a scenario of two secants intersecting a circle. The following equation can be written.

The measure of angle E is equal to one half the quantity eighty eight minus twenty six.

This equal one half of sixty two or thirty one degrees.

**Slide 4:**

Let's do another example together.

Given that the measure of arc LN equals 145 degrees and the measure of arc NK equals 130 degrees, find the measure of angle M.

First of all, it is important to note which arc measurements are given. The property we learned involved the intercepted arcs. In this case, we need to know arc LN and arc LK. We do not know arc LK. To find arc LK, we will take 360 and subtract the measure of arc LN and subtract the measure of arc NK. This gives us 360 minus 145 minus 130, which equals 85 degrees.

Now, the property states that the measure of an angle when two lines intersect outside the circle is equal to one-half the difference of the measures of the intercepted arcs. In this case, the measure of angle M equals one-half the difference of the measure of arc LN minus the measure of arc LK. Using the measures given, we find one-half the difference of 145 minus 60, which simplifies to 30. The measure of angle M equals 30 degrees.

**Slide 5:**

Now you try.

Answer the following questions. Click on solution to check your work.

1. Find the measure of angle B. The circle has point B outside the circle, segment BA tangent to the circle, and secant from B, through points C and D on the circle. The measure of arc AD equals 85 degrees. The measure of arc AC equals 25 degrees.

**Solution**

The measure of angle B equals one-half the quantity 85 minus 25 equals 30 degrees.

2. Find the value of  $x$ . The circle has point X outside the circle, segment XY tangent to the circle, and secant from X, through points Z and W on the circle. The measure of arc WY equals  $25x + 2$ . The measure of arc YZ equals  $17x$  and the measure of angle X equals 17 degrees.

**Solution:**

The measure of angle X equals one-half the quantity the measure of arc WY minus the measure of arc YZ.  $17$  equals one-half the quantity  $25x + 2$  minus  $17x$ .  $17$  equals one-half the quantity  $8x + 2$ .  $17$  equals  $4x + 1$   $16$  equals  $4x$ .  $4$  equals  $x$ .

**Slide 6:**

**Remember**

When tangents and secants intersect outside a circle, the angle of intersection can be found by taking one-half the difference of the intercepted arcs.

Be careful that you use the correct arcs when setting up the calculations.