

Tutorial: Properties of Chords**Slide 1:**

In this tutorial, we are going to learn about the following: intersecting chords, distance a chord is from the center of the circle, and chords perpendicular to a segment from the center of the circle.

Slide 2:

Let's first look at Intersecting Chords.

When chords intersect, four segments are created. The product of the lengths of the two segments from one chord is equal to the products of the lengths of the two segments from the other chord.

Notice in the circle shown, that one chord is made up of a segment with length "a" and a segment with length b. The second chord is made up of a segment with length x and a segment with length y. A times b equals x times y.

Slide 3:

Let's look at an example of intersecting chords. Find the value of p in the figure below.

The circle has two chords. The first has segments with length 4 and length 3. The second has segments with length 5 and length p minus 1.

Remember the property that we just learned. 4 times 3 equals 5 times the quantity p minus 1. Simplify to get 12 equals 5p minus 5. Add 5 to both sides to get 17 equals 5p. Divide both sides by 5. This means that 3.4 equals p.

Slide 4:

Let's look at chords and their distance from the center of the circle. The distance is measured by a perpendicular segment from the center.

Two chords in the same circle are the same distance away from the center of the circle if and only if they are the same length.

If the red segment is the same length as the blue segment, then their distances from the center are equal. Also, if their distances are equal, then the red segment has the same length as the blue segment.

Slide 5:

Let's look at an example.

What is the value of x in the figure below?

Notice that the distances of the chords from the center are equal, they both are 3. This means that the two chords have equal length.

$3x - 4 = 8$. Write this equation and solve it. $3x = 12$, which means x equals 4.

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Now let's look at chords which are perpendicular to a segment in a circle. Specifically, the segment we will look at is one that passes through the center of the circle.

A chord is perpendicular to a segment that goes through the center of a circle, if and only if that chord is bisected.

Take a look at the segment from the center of the circle and notice that it is perpendicular. According to this property, we then know that the red portion of the chord has the same length as the blue portion of the chord.

Looking at this the other way, if we KNOW that the red portion has the same length as the blue portion, we know that the segment from the center of the circle is perpendicular to the chord.

Slide 7:

Let's look at an example.

Notice that in the circle, we have a chord that intersects a segment drawn from the center of the circle. The segment is perpendicular to the chord. This means that it bisects it. The red segment has the same length as the blue segment.

10 equals $6x$ minus 14. Solve this equation.

Add 14 to both sides to get 24 equals $6x$ and then divided by 6 to get 4 equals x .

Slide 8:

Now you try. Answer each of the following questions. Click on Solution to check your work.

1. What is the value of a in the figure below? The figure has two intersecting chords. The segments made in the first chord are 5 and a minus 1. The segments made in the second chord are a plus 1 and 3.

Solution:

5 times the quantity a minus 1 equals 3 times the quantity a plus 1.

$5a$ minus 5 equals $3a$ plus 3.

$5a$ equals $3a$ plus 8.

$2a$ equals 8.

A equals 4.

2. What is the value of b in the figure below? The figure has two chords of length 15. One chord has a perpendicular segment from the center of the circle with length $3b$ minus 6. The second chord has a perpendicular segment from the center of the circle with length b plus 5.

Solution:

Since the chords are the same length, they are the same distance from the center.

$3b$ minus 6 equals b plus 5.

$3b$ equals b plus 11.

$2b$ equals 11.

3. What is the value of c in the figure below? The figure has a chord with two segments of length 12. The radius that intersects the chord makes an angle that is labeled $2c$ plus 22 degrees.

Solution:

Since the chord is bisected, it must be perpendicular to the segment from the center of the circle.

$2c$ plus 22 equals 90.

$2c$ equals 68.

C equals 34.

Slide 9:

Remember: If two chords intersect, the product of the lengths of the two segments from one chord is equal to the product of the lengths of the two segments from the other chord, two chords in the same circle are the same distance away from the center of the circle if and only if they are the same length, and a chord is perpendicular to a segment that goes through the center of a circle, if and only if that chord is bisected.