

**Example: Properties of Secants****Problem:**

Find the value of  $x$  in the figure below. It is a circle with two secant lines drawn from the same point outside the circle. One secant line has the outside segment with length 2.5 and the inside segment with length 3.9. The other secant line has the outside segment with length  $x$  and the inside segment with length 6.

**Solution:**

Recall that when secant lines intersect outside the circle, the segments are related in a particular way. The length of the outside segment times the length of the whole segment of one secant is equal to the length of the outside segment times the length of the whole segment of the other secant. In this case, this means  $x$  times the sum  $x + 6$  is equal to 2.5 times the sum 2.5 plus 3.9.

Distribute and simplify to get  $x^2 + 6x = 2.5(2.5 + 3.9)$ .

$x^2 + 6x = 16$ .

Subtract 16 from both sides to get  $x^2 + 6x - 16 = 0$ .

At this point, you can either use the quadratic formula or you can factor. This one is fairly easy to factor.

The quantity  $x - 2$  times the quantity  $x + 8$  equals 0.

Since the product of two factors is zero, we can set each factor equal to zero.

$x - 2 = 0$  OR  $x + 8 = 0$ .

Solve these to get  $x = 2$  OR  $x = -8$ .

Since  $x$  is a length of a segment, the negative answer does not make sense. The final answer is  $x = 2$ .