

Example: Properties of Kites**Problem:**

Quadrilateral ABCD is a kite. Find the measure of angle ABC and the length of side BC if you are given that the measure of angle BAD equals 90 degrees, the measure of angle BCD equals 42 degrees, DB equals 12, AB equals 10, and AC equals 18.

Solution:

We are told that angle BAD equals 90 degrees, and that angle BCD equals 42 degrees. Since this is a kite, we know that angle B and angle D are congruent. We will use the variable 'x' for those angle measures.

The sum of the angles in a quadrilateral is 360. 90 plus 42 plus $2x$ equals 360 . Solve this equation for x .

132 plus $2x$ equals 360 .

$2x$ equals 228 , therefore x equals 114 .

The measure of angle ABC is 114 degrees.

Let's now look at the segment lengths.

We know that the diagonal between the two congruent angles is bisected by the other diagonal. XB is one-half of DB. One-half of 12 is 6 .

Now, because the diagonals are perpendicular, a right triangle is created.

Segment AB is 10 and segment XB is 6 .

Since this is a right triangle, we can use the Pythagorean Theorem. AX squared plus XB squared equals AB squared.

AX squared plus 6 squared equals 10 squared.

Solve this equation to find that AX equals 8 .

Remember that we are trying to find the length of side BC. We have a little more work to do.

AX plus XC equals AC.

8 plus XC equals 18 , which means that segment XC equals 10 .

Again, because the diagonals are perpendicular, a right triangle is created.

Segment XB is 6 and segment XC is 10 .

Since this is a right triangle, we can use the Pythagorean Theorem. XB squared plus XC squared equals BC squared.

6 squared plus 10 squared equals BC squared.

Solve this equation to find that BC equals 11.66 .