**Course: Geometry**
**Unit: Introduction to Proof**
**Section: Reasoning in Geometry**

**Tutorial: Truth Tables**

**Slide 1:**
In this tutorial, you will learn about constructing truth tables and evaluating logical statements.

**Slide 2:**
A simple statement can either be true or false, but not both.
Consider the statement:
If two angles are alternate interior angles, then they are congruent.
The hypothesis, p, is that two angles are alternate interior angles,
the conclusion, q, is that they are congruent.

We can create a truth table to organize the results. Truth tables can be simple, or they can be quite complex.

**Slide 3:**
Look at this example of a truth table for the negation of statement p.
In the first column, we place statement p, which is that two angles are alternate interior angles.
In the second column, we have the negation, not p, which would be that two angles are not alternate interior angles.
If p is true then not p is false.

For our statement, if "two angles are alternate interior angles" is a true statement, then "two angles are not alternate interior angles" would be a false statement.

Likewise, if p is false then not p is true

If "two angles are alternate interior angles" is a false statement, then "two angles are not alternate interior angles" would be a true statement.

**Slide 4:**
For the conditional statement, p implies q, there will be more rows and columns in our truth table.
Consider the truth table below for the conditional statement p implies q. If p is true and q is true, then p implies q is true.
If p is true, and q is false, then p implies q is false.
This would mean that if two angles are alternate interior angles and they were somehow not congruent, then p implies q would not be true. This is would perhaps be a situation you could see in non-Euclidean geometry.
If p is false, and q is true, then p implies q is true.
This may be confusing. It simply means that q is true no matter what p is. If you think of p implies q as an input-output rather than as a cause and effect, it may make more sense.
If p is not true, but q is true no matter what, then p being false does not affect p implies q.

If p is false, and q is false, then p implies q is true.
For our example, if two angles are not alternate interior angles and they are not congruent, then one statement does imply the other.

The truth table would look the same no matter what our statements were.

**Slide 5:**
We can also evaluate the truth of the converses, inverses, and contrapositives of our statements.

From our statements, p and q, we can write the conditional statement "If two angles are alternate interior angles, then they are congruent.

The converse is "If two angles are congruent, then they are alternate interior angles." Can you think of a counterexample?

Right angles are congruent, and are not necessarily alternate interior angles, so the converse is false.

The Inverse is "If two angles are not alternate interior angles, then they are not congruent" Is this true? Not always. Just because two angles are not alternate interior angles does not mean that they cannot be congruent. The inverse is false.

Finally, what about the contrapositive? "If two angles are not congruent, then they are not alternate interior angles." We know that if two angles are not congruent, then they cannot be alternate interior angles. The contrapositive is true.

**Slide 6:**

Consider the conditional statement, "If two numbers are odd, then their sum is even." Choose whether each statement is true or false to complete the table below.

P implies q: True. This is a true conditional statement.

Q implies p: False. The sum of two even numbers is also even.

Not p implies not q: False: If the two numbers are even, then their sum is also even.

Not q implies not p: True. If the sum of two numbers is not even, then the two numbers can not be odd.

**Slide 7:**

In this tutorial, you learned about constructing truth tables. You also learned about evaluating logical statements including converses, inverses, and contrapositives.