i **Imaginary and Complex Numbers**

Since i is defined to have the property that $i^2 = -1$, the number i is the principal square root of -1; that is, $i = \sqrt{-1}$. i is called the imaginary unit. Numbers of the form $3i$, $-5i$, and $i\sqrt{2}$ are square roots of negative real numbers. For any positive real number b, $\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1}$ or bi.

Example 1) $\sqrt{-18} = \sqrt{(-1)(3^2)(2)} = \sqrt{-1} \bullet \sqrt{3^2} \bullet \sqrt{2} = 3\,i\sqrt{2}$

Example 2) $-2i \bullet 7i = -14i^2 = -14(-1) = 14$

Example 3) $\sqrt{-10} \bullet \sqrt{-15} = i\sqrt{10} \bullet i\sqrt{15} = i^2\sqrt{150} = -1\sqrt{25} \bullet \sqrt{6} = -5\sqrt{6}$

Simplify:

1.	$\sqrt{-144}$	$12i$	6.	$3i(-5i)^2$	$-75i$
2.	$\sqrt{-64x^4}$	$8x^2i$	7.	i^{38}	-1
3.	$\sqrt{-6} \bullet \sqrt{-24}$	-12	8.	$\sqrt{-120}$	$2i\sqrt{30}$
4.	$(-2i)(-6i)(4i)$	$-48i$	9.	$-2\sqrt{-168}$	$-4i\sqrt{42}$
5.	i^{24}	1	10.	$\sqrt{-125x^5}$	$5ix^2\sqrt{5x}$

Complex numbers

A complex number is any number that can be written in the form a + bi, where a and b are real numbers and i is the imaginary unit. For example, $7 + 4i$ and $2 - 6i$ are complex numbers.

Complex numbers can be added and subtracted as if they were binomials.

Example 1) $(3 + 4i) + (2 + 7i) = 3 + 2 + 4i + 7i = 5 + 11i$

Example 2) $(-5 + 8i) - (2 - 12i) = -5 - 2 + 8i + 12i = -7 + 20i$

The numbers a + bi are multiplied as if they were binomials, with $i^2 = -1$.

Example 3) $(3 + 4i)(2 + 7i)$
$= 6 + 21i + 8i + 28i^2$
$= 6 + 21i + 8i + 28(-1)$
$= 6 - 28 + 29i$
$= -22 + 29i$

Example 4) $(-2+\sqrt{-16})(4-\sqrt{-9})$
$= (-2 + 4i)(4 - 3i)$
$= -8 + 6i + 16i - 12i^2$
$= -8 + 12 + 22i$
$= 4 + 22i$

Simplify:

11. $(6 - 2i)(1 + i)$ $8 + 4i$

12. $(3 - 5i)(3 + 5i)$ 34

13. $(1 - 4i)(2 + i)$ $6 - 7i$

14. $(7 + \sqrt{-36})(6 + 4i)$ $18 + 64i$

15. $(10 - \sqrt{-25})(-15 - \sqrt{-49})$ $-185 + 5i$

Two complex numbers of the form a + bi and a – bi are called complex conjugates. The product of complex conjugates is always a real number. For example, $(2 + 3i)(2 - 3i) = 4 - 6i + 6i + 9$ or 13. You can use that fact when simplifying the quotient of two complex numbers. When dividing complex numbers it is important to remember that you must not leave the *i* in the denominator.

Example 1) $\frac{3i}{2+4i} = \frac{3i}{2+4i} \bullet \frac{2-4i}{2-4i} = \frac{6i-12i^2}{4-16i^2} = \frac{6i+12}{20} = \frac{3i+6}{10}$

Example 2) $\frac{6}{2i} = \frac{6}{2i} \bullet \frac{i}{i} = \frac{6i}{2i^2} = \frac{6i}{2(-1)} = \frac{6i}{-2} = -3i$

<u>Simplify:</u>

16. $\frac{4}{5+3i}$ $\frac{10-6i}{17}$

17. $\frac{3}{4i}$ $-\frac{3i}{4}$

18. $\frac{2-i}{3-4i}$ $\frac{2+i}{5}$

19. $\frac{10+i}{4-i}$ $\frac{39+14i}{17}$

20. $\frac{8}{3i}$ $-\frac{8i}{3}$