Convergence Tests - Summary and Study Chart

Here is a summary of all the convergence tests that we have used in this chapter. I have tried to list them in order that you should use/try them when testing for convergence or divergence.

For **Series** (a sum of terms) there are many tests that can be used. Remember that **you always must name the test and show all steps when using these**. Simply stating that the series is convergent or divergent will not earn you full points. **The only exception is when you have the harmonic series or alternating harmonic series**. If you name the series as one of these, then you can simply say something like "divergent harmonic series".

Convergence Tests for Series	Comment
n^{th} Term Test $\sum_{n=1}^{\infty} a_n$ diverges if $\sum_{n=1}^{\infty} a_n \neq 0$	This test only proves divergence. If $\sum_{n=1}^{\infty} a_n$ converges, then a_n must approach 0, but this test cannot prove convergence.
Geometric Series with ratio, r: $\sum_{n=0}^{\infty} a\,r^n$ if $\lvert r \rvert < 1$, the series converges if $\lvert r \rvert \geq 1$, the series diverges The sum of an infinite geometric series can be found: $S = \frac{a}{1-r}$.	You have to show that the series converges before you can use the formula to find the sum!
p- Series: $\sum_{n=0}^{\infty} \frac{1}{n^p}$ if $p > 1$, the series converges if $p \leq 1$, the series diverges	
Telescoping Series: $\sum_{n=1}^{\infty} \left(b_n - b_{n+1}\right)$ These are normally expressed with partial fractions. Expand and simplify to find the sum.	Don't be fooled into thinking the sum will always equal the first term. Always expand and check to see which terms are canceling out.
Alternating Series Theorem The series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ will converge if both of the conditions below are met:. a) $a_n \geq a_{n+1}$ for all n (the terms decrease in size) b) $\lim_{n\to\infty} a_n = 0$ (the terms approach zero)	If you are asked to find if your alternating series converges absolutely, check the absolute value series first. If it does converge, then you are done – just state the conclusion. If not, test the alternating series with the AST for conditional convergence. **You do not have to** test for absolute convergence unless you are asked to do so!
Direct Comparison Test Compare a series to a series whose behavior we already know. If a series is always less than a series that converges, then it too must converge. If it is always greater than a series which diverges, then it will also diverge.	Be sure to show (explain) why your comparison series converges or diverges.
Limit Comparison Test If the terms of two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are positive for $n \geq N$, and $\lim_{n\to\infty} \frac{a_n}{b_n}$ is finite and positive, then either both series converge or both diverge.	Be sure to show (explain) why your comparison series converges or diverges. This test often works when the Direct Comparison does not.
Integral Test If f(x) is a continuous, positive, and decreasing function	Be sure to show that the function is continuous, positive, and decreasing before using the test.

of x for all $x \ge 1$, then the series $\sum_{n=1}^{\infty} a_n$ and the integral $\int_1^{\infty} f(x)\,dx$ will both converge or both diverge.	Remember that the integral value will not necessarily be the same value as the limit of the series or the sum of the series.
Ratio Test If $\sum a_n$ is a series with positive terms, where $\lim_{n\to\infty}\left\|\frac{a_{n+1}}{a_n}\right\| = \rho$. then: a) the series converges if $\rho < 1$. b) the series diverges if $\rho > 1$. c) no information is provided if $\rho = 1$.	Make sure you only use the absolute value of the terms when you find the ratio! If the limit is equal to one, you have to try another test; you can't just say "inconclusive" and stop.
Root Test Let $\sum a_n$ be a series with positive terms, where $\lim_{n\to\infty}\sqrt[n]{\|a_n\|} = \rho$. then: a) the series converges if $\rho < 1$. b) the series diverges if $\rho > 1$. c) no information is provided if $\rho = 1$.	Make sure you take the n^{th} root of the absolute value of a_n. If the limit is equal to one, you have to try another test; you can't just say "inconclusive" and stop.
Testing a Power Series for Convergence 1. Use the Ratio or n^{th} Root Test to find the interval where the series converges absolutely. Take the limit of the ratio or root and then put it between -1 and 1 to find the interval of absolute convergence. 2. If the interval of convergence is finite, also check the endpoints. Use a Comparison Test, the Integral Test, or the Alternating Series Theorem to test the endpoint series. 3. Give the final interval of convergence.	If the power series is geometric, there is no need to test the endpoints since the absolute value of the ratio of a convergent geometric series must be between 1 and -1. Be sure to label which endpoint you are checking and show your work and conclusion for each endpoint.