Newton's Law of Cooling

Newton's Law of Cooling deals with the rate at which an object will change temperature when brought into a new environment of constant temperature. The law is:

\[
T = T_s + (t_0 - T_s) e^{-kt}
\]

T is the temperature of the object at time t,
\(T_s\) is the surrounding temperature of the environment,
t_0 is the initial temperature of the object.

Notice that, just like in previous equations, the sub 0 always designates initial conditions. Also, you will not have to memorize this formula. If you need it, I will give it to you.

These problems are not bad but they usually do take a bit of algebra to get to the answer. Let’s do a problem using this formula.

In this problem a pan of warm (46° C) water is put into a refrigerator to cool down. At \(t = 10\), \(T = 39° C\) and 10 minutes later (at \(t = 20\)), \(T = 33° C\). We need to find how cold the refrigerator is, \(T_s\).

First we will use the data given to set up two equations, using the given information about time and temperature.

\[
\begin{align*}
39 &= T_s + (46 - T_s) e^{-10k} \\
33 &= T_s + (46 - T_s) e^{-20k}
\end{align*}
\]

I am going to solve for \(e^{-10k}\) in the first equation.

Now I am going to put this in the second equation since the square of \(e^{-10k}\) is \(e^{-20k}\). Simplify.

Multiply through by the \(46 - T_s\). Square the binomial on the right side.

Simplify. Notice how that horrible-looking algebra problem turned out nicely! We just had to keep plodding along carefully.