Initial Conditions and Finding the Particular Solution

Let's review the subject of **Initial Conditions.** **This is where we are given some information which will allows us to find the value of the mystery constant**. When we only find the antiderivative with the + C, we have a family of functions that all differ by a constant. With initial conditions we find a single solution that belongs to a unique function.

Here are some examples:

A)

dy/dx = 2x - 7 y= 0 when x = 2	This is the differential equation. Initial conditions have been given.
$y = x^2 - 7x + C$	First take the antiderivative of dy/dx = 2x - 7. Remember that you can check by taking the derivative of the answer and comparing with the derivative you were given to start with.
y = 0 when x = 2: 0 = 4 - 14 + C C = 10	Sub in the initial condition values you were given and solve for C. **Make sure you write down those initial conditions so that your work is easy to follow.**
$y = x^2 - 7x + 10$	Finally, rewrite the function with the value of C. **You must do this step for full credit**.

We can do a quick check to see if we got the right answer.

1 – Take the derivative of $y = x^2 - 7x + 10$. It is in fact the derivative which we were given: dy/dx = 2x - 7.
2 – Put in the point we were given and see if it works: 0 = 4 – 14 + 10 = 0 it does!

B) If we are given velocity and asked to find the equation for position, we have to take the antiderivative. Remember that the derivative of position is velocity and the derivative of velocity is acceleration.

<u>Derivatives</u> move us to the right from **position -> velocity -> acceleration**. The <u>antiderivative will move us back to the left.</u>

v = 9.8t + 5 s = 10 when t = 0	This is the derivative function for velocity. Initial conditions have been given.
$s = 4.9t^2 + 5t + C$	First take the antiderivative. Remember that the antiderivative of velocity is position.
s = 10 when t = 0 10 = 0 + 0 + C C = 10	Sub in the values you were given and solve for C.
$s = 4.9t^2 + 5t + 10$	Finally, rewrite the function with the value of C. A quick derivative check verifies our work: s ‘ = 9.8 t + 5 and if we put in t = 0 and s = 10, we get 10 = 0 + 0 + 10 so the initial conditions work too.

C) We are given a fourth derivative and have to find our way back to the original function. This means we will be taking the antiderivative 4 times, **stopping each time to find the C.** This last bit is very important! Don't clog your work up with tons of constants, because each time you take the antiderivative you will get a new one. It is much easier to **stop each time and find the value of the constant, and then move on to the next step.**

$y^{(4)}$ = - sin t + cos t	This is the fourth derivative function.
y ''' = 7, y '' = -1, y ' = -1, and y = 0 when t = 0	Initial conditions have been given. Lots of them because we are to find the original function y, which is 4 steps back! We will have to pay careful attention to the notations so that we use the right values for each step.
y ''' = cos t + sin t + C y ''' = 7 when t = 0: 7 = cos 0 + sin 0 + C 7 = 1 + 0 + C C = 6 y ''' = cos t + sin t + 6	Find the first antiderivative. We stop and find the value of the first mystery constant (constant of integration is its official name). I put the 7 in for y ''', and the 0 in for t. Do not forget to stop and do this **every time** you go through the process of finding the antiderivative. Write the entire third derivative before moving on.
y '' = sin t - cos t + 6t + C y '' = -1 when t = 0 -1 = sin 0 - cos 0 + 0 + C -1 = 0 - 1 + 0 + C C = 0 y '' = sin t - cos t + 6t	This is the second antiderivative. Again we find the new constant of integration, using the values for t and y '' that we were given in the initial conditions. Write the entire second derivative before moving on.
y ' = - cos t - sin t + $3t^2$ + C y ' = -1 when t = 0 -1 = -cos 0 - sin 0 + 0 + C -1 = -1 - 0 + 0 + C C = 0 y ' = -cos x - sin t + $3t^2$	This is the third antiderivative. Again we find the new constant of integration, using the values for t and y ' that we were given in the initial conditions. Only one more time to go!
y = - sin t + cos t + t^3 + C y = 0 when t = 0: 0 = - sin 0 + cos 0 + 0 + C 0 = 0 + 1 + 0 + C C = -1 y = - sin t + cos t + t^3 - 1	This is the fourth and last antiderivative. Again we find the new constant of integration, using the values for t and y ' that we were given in the initial conditions. Now we are done. We have found y, the original function using antidifferentiation and the given initial conditions.

One last note: You can always check to see if your antiderivative is correct; just take the derivative of your answer and see if you end up with what you were given to start. It is easy to do and well worth doing! Also, you can put your initial conditions back into the final answer and see if they work with the value of C that you came up with. This is a good way to check for sign errors that you might have made when solving for C.

I have one other example I want to do. As you go through all of these, **if you don't understand something, let me know right away.** It is your responsibility to let me know when something does not make sense - my ESP is dismal.

D) We are to find the curve in the xy-plane that passes through the point (9,4) and whose slope at each point is $3\sqrt{x}$.

$m_t = 3\sqrt{x}$ $\frac{dy}{dx} = 3x^{1/2}$	First we have to translate the second part of the sentence into math. **We know the slope of a tangent line or the curve comes from the first derivative of the equation of the curve, so I have rewritten the equation with derivatives.**
$y = 3\left(\frac{2}{3}x^{3/2}\right) + C$ $y = 2x^{3/2} + C$ through (9, 4): $4 = 2(9)^{3/2} + C$ $4 = 54 + C$ C = -50	Take the antiderivative. Adding one to 1/2 gives a new exponent of 3/2. Use our initial conditions to find C: We have a point (9, 4) so we know that x = 9 and y = 4.
$y = 2x^{3/2} - 50$	Finally we write the final equation, with the value of C included.

Not too bad, I hope. Just be careful to do a quick check when you are done (especially with the trig functions) and to stop to use your initial conditions to find the constant of integration every time you take the antiderivative.